

## PROSPECTS FOR AN IMPROVED SUPERCONDUCTING CAVITY STABILIZED OSCILLATOR CLOCK

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### ABSTRACT

Superconducting Cavity Stabilized Oscillators (SCSO) have produced the most stable clocks to date for integration times between  $10^2$  and  $10^3$  seconds, achieving an Allen variance of  $3 \times 10^{-16}$ . The principal contributors to cavity frequency variations are: a) acceleration effects due to gravity and vibrations b) temperature variations, and c) variations in the energy stored in the cavity. We discuss the prospects for improvements in all these areas, for both ground and space based SCSO's. Improvements of at least one order of magnitude should be achievable on the ground, while space offers the opportunity for a  $10^{-18}$  clock for the range around  $10^3$ - $10^4$  seconds integration time. We also show that calculations of cosmic radiation effects indicate that the frequency variations induced by heating and trapped flux generation, annihilation and motion are smaller than  $10^{-18}$ . Finally we discuss some of the possible ground and space based applications of an improved SCSO clock. These include the measurement (in conjunction with hydrogen masers) of the dependence of fundamental constants on the gravitational potential, gravitational redshift measurements, measurements of the anisotropy of the velocity of light, and the potential for gravity wave detection.

## I. INTRODUCTION

Superconducting Cavity Stabilized Oscillators (SCSO's) have achieved stabilities of a few parts in  $10^{16}$  for periods of around 100 s, making them the most precise clocks in this interval range. Stein and Turneaure<sup>1)</sup> have demonstrated Allan variances of  $3 \times 10^{-16}$  for solid 8.6 GHz  $TM_{010}$  niobium cavities with unloaded quality factors of up to  $2 \times 10^{11}$ . More recent work using superconductor coated sapphire resonators<sup>2),3)</sup> have reached short term frequency stabilities between  $10^{-15}$  and  $10^{-14}$ . We discuss the prospects of improving the stability of SCSO's by a factor of more than 30 for Earth based experiments, and a factor of more than 100 by taking advantage of the low gravity and low disturbance environment of space based applications.

Possible applications for clocks of this type and quality are: a) various experimental checks of gravitational theories, b) measurements of the time and gravitational potential dependence of fundamental physical constants, c) verification of the isotropy of the velocity of light, and d) possible detection of gravitational waves. In addition SCSO's coupled as flywheels with atomic standards would create accurate clocks of very high stability for time ranges above 0.1 s.

Vessot<sup>4)</sup> has proposed a number of clock experiments aimed at verifying Einstein's theory of General Relativity. First, an improved measurement of the PPN parameter<sup>5)</sup>  $\beta$  is possible, by measuring the second order gravitational redshift seen by a clock in a polar Solar orbit. The present  $\sim 0.1\%$  accuracy of  $\beta$ , obtained from null measurements of the Nordtvedt effect,<sup>6)</sup> could be improved by two orders of magnitude by flying a high stability SCSO. Second, an SCSO flying in an Earth eccentric orbit could be used to make a gravitational redshift measurement to better than  $10^{-6}$ , also a two orders of magnitude improvement over the present results.

The best null gravitational redshift experimental accuracy is currently about 2%, measured in a ground-based comparison of an ensemble of SCSO's with hydrogen masers<sup>7)</sup> in the diurnal and yearly varying gravitational potential on Earth. A comparable experiment performed with one of the clocks in an eccentric Earth orbit, and assuming an atomic clock with stability matching that of the SCSO, would yield a test of the Einstein Equivalence Principle to the  $10^{-9}$  level. Similar tests would put stringent limits on the variation with time and with gravitational potential of the fundamental physical constants.

Anisotropies in the velocity of light are detectable as variations of the SCSO frequency with respect to the orientation of the local frame relative to the microwave cavity and its  $TM_{010}$  fields. Gravitational waves distort the cavity and could be detected by a SCSO with frequency stability better than  $10^{-18}$ .

## II. DESCRIPTION OF THE SCSO CLOCK

The microwave resonator is a solid superconducting niobium cavity operating at 1.3 K, in the  $TM_{010}$  mode, at 8.6 GHz, and under ultrahigh vacuum conditions. In order to insure mechanical stability the walls of the cavity are of about the same thickness as its 1.3 cm radius.

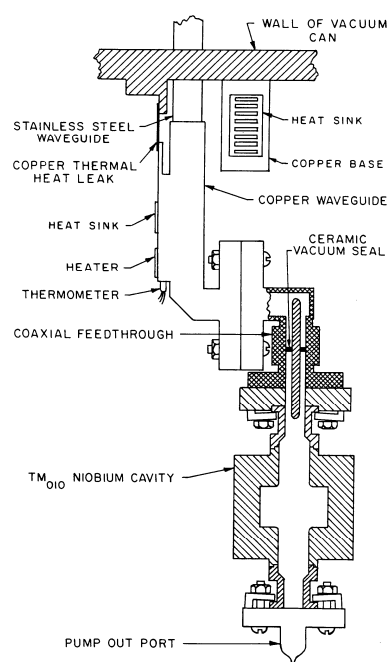


Figure 1. Schematic of SCSO resonator and its mounting.

Figure 1 shows a schematic representation of the cavity and its mounting scheme in a vacuum can that is immersed in pumped liquid helium. The cavity is mounted with support from the top and it is connected with indium sealed vacuum flanges to the pump-out port and to the microwave waveguide. High vacuum conditions for the cavity are maintained by means of a permanent internal vacuum with a pinch-off and by the exterior vacuum can. The temperature of the cavity is controlled to  $1 \mu\text{K}$  short term and  $10 \mu\text{K}$  per week. Magnetic shields insure that the field at the cavity is less than 10 mG. Both the dewar and the electronics are tilt controlled to reduce the effect of variations in local gravity. The entire apparatus consisting of three SCSO systems is cooled in a top loading dewar.

Special care is taken in preparing the resonator cavity surfaces in order to achieve the highest  $Q$  values. The cavity is machined of ultra pure niobium and then undergoes a conditioning process consisting of an initial chemical cleaning, a  $1900^\circ\text{C}$  ultrahigh vacuum firing, a chemical etching of a  $100 \mu\text{m}$  surface layer, and a final  $1900^\circ\text{C}$  ultrahigh vacuum firing. Typical values for the unloaded  $Q$  factor obtained by this method are about  $10^{11}$ , with the best cavities exhibiting a  $Q$  in excess of  $2 \times 10^{11}$ . Surface effects and not the temperature are presently limiting the achievable  $Q$ .<sup>8)</sup>

The design of the oscillator is based on utilizing the high Q cavity resonance to stabilize a voltage controlled oscillator. In the present design half of the power of a Gunn oscillator with variable frequency is used to excite the cavity. This signal is phase modulated at 1 MHz and the AM modulated signal reflected by the cavity is then detected. The sign and amplitude of the detected AM signal represent the deviation of the Gunn oscillator frequency from the cavity frequency, and are used to servo the Gunn oscillator. Figure 2 shows a block diagram of the SCSO developed at Stanford University. The connection of the resonator to room temperature is made using stainless steel waveguide with copper baffles in order to minimize thermal losses.

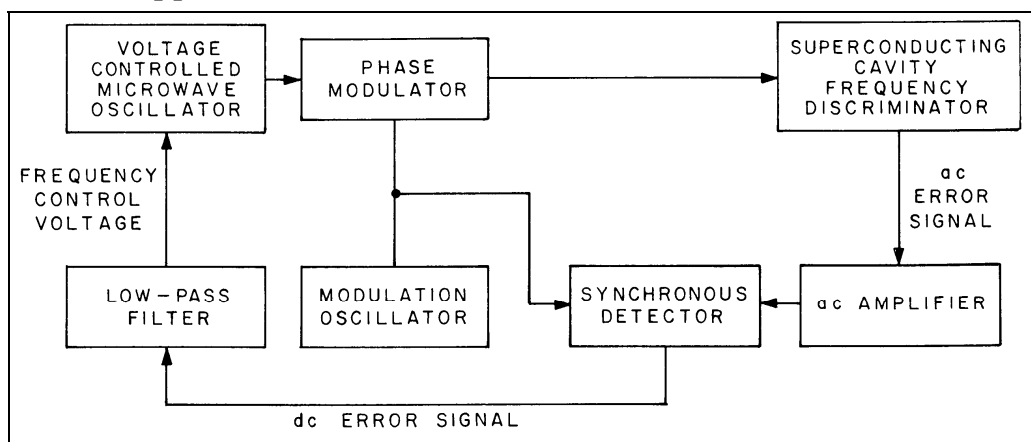


Figure 2. Block diagram of SCSO

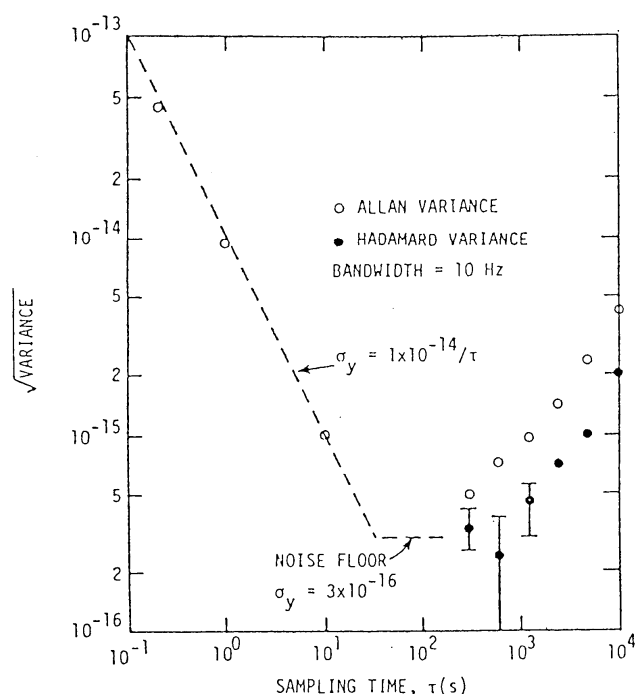


Figure 3. Allan and Hadamard variances for the 8.6 GHz SCSO.

The Allan and Hadamard variances, measured with a 10 Hz noise bandwidth, are plotted in Figure 3 as a function of the sampling time  $\tau$ . For  $\tau < 30$  s,  $\sigma_y = 10^{-14}/\tau$ . The noise floor of  $3 \times 10^{-16}$  is reached for  $30 \text{ s} < \tau < 1000$  s. For  $\tau > 1000$  s, the fractional long term drift of the SCSO is about  $2 \times 10^{-13}/\text{day}$ . An ensemble of three SCSOs were used to measure the frequency stability of the device. Combined with two hydrogen masers this ensemble also yielded a 2% level confirmation of the independence of the hyperfine constant  $\alpha$  from the variation of a weak gravitational potential.

### III. MAIN DISTURBANCE EFFECTS AND PROPOSED IMPROVEMENTS

We consider the strategies for improving both the noise floor and the long term stability of the SCSOs. For reducing the noise floor we discuss improvements in the electronics of the system. Long term stability can be improved by reducing the main disturbance effects, i.e. local gravity variations, temperature fluctuations, and variations in the energy stored in the cavity. In addition we discuss the frequency variations due to the effects of cosmic radiation. A number of smaller disturbance effects including quantum fluctuations, thermally induced phonons, external pressure variations, and low temperature structural changes (creep) are estimated to cause frequency instabilities below the  $10^{-18}$  level, and will therefore not be discussed.

#### A. ELECTRONIC CIRCUIT

Conceptually the scheme for the SCSO electronics will quite similar to the original design. However, some major changes to the circuit design will be incorporated based on the following principles: a) use of improved microwave technology which was not available for the original circuit, b) a stabilizing power servo, and c) extensive use of cryogenic components. The Gunn oscillator will be replaced with a varactor-tuned

dielectric resonator oscillator (DRO) selected for very low close-in phase noise. The oscillator will be placed in the vapor cooled region of the cryostat, at around 6 K, and will be temperature controlled to  $10^{-2}$  K. An intermediate temperature control stage at about 1.5 K, and with a thermal stability of  $10^{-4}$  K, will provide the cryogenic environment for the phase modulator, the power stabilization servo, and the AM detector. Power stability of better than 0.1% should be easily achievable. We expect to be able to reduce the noise floor below the goal of better than  $10^{-17}$  frequency stability level.

### B. LOCAL ACCELERATION AND GRAVITY

Vibrations and variations in local gravity will change the frequency by changing the dimensions of the cavity. For the TM<sub>010</sub> mode vertically mounted cavity the frequency is dependent to first order only on the average diameter of the resonator, and thus only sensitive to second order in the tilt angle. The frequency stability  $|\delta\nu/\nu_0|$  is:

$$\left|\frac{\delta\nu}{\nu_0}\right| \cong \left|\frac{\delta R}{R}\right| \cong \frac{1}{3} \cdot \frac{\delta l}{l} \cong \frac{1}{3} \cdot \frac{l\rho(\delta g)}{Y} \Rightarrow \left|\frac{\delta\nu}{\nu_0}\right| \cong 4 \times 10^{-9} \cdot \frac{\delta g}{g} \quad (1)$$

where  $R$ ,  $l$ ,  $\rho$ , and  $Y$  are the radius, length, density and Young modulus of the cavity. Earth tides are easily observable at the  $10^{-14}$  frequency variation level, in full agreement with the model above.

The sensitivity to variations in the local field can be reduced significantly for ground based experimentation by supporting the cavity from its center, consequently compensating any change in the length of the top half with the opposite change in the length of bottom half. Furthermore, the connection to the waveguide will be made via a choke flange joint, thus leaving the center support as the only mechanical connection to the resonator. The expected improvement from the center support will increase the frequency stability under variations of the local gravity to:  $|\delta\nu/\nu_0| \cong 4 \times 10^{-11} \cdot \delta g/g$ . Space experiments will benefit from an environment in which  $\delta g/g \leq 1 \times 10^{-8}$ , therefore making this effect negligible. Frequency instabilities due to variations in the local  $g$  will thus be reduced below the  $10^{-18}$  level.

### C. TEMPERATURE FLUCTUATIONS

Fluctuations in temperature affect the frequency stability via two main effects on the cavity: thermal expansion and the variation with temperature of the skin depth. At 1.2 K these two effects are approximately equal:

$$\left| \frac{\delta\nu}{\nu_0} \right| = B \cdot \exp\left(\frac{-C \cdot g(T)}{T}\right) + D \cdot T^E \cong 9 \times 10^{-6} \cdot \exp\left(\frac{-17}{T}\right) + 1 \times 10^{-11} \cdot T^4 \quad (2a)$$

$$\frac{\partial \left| \delta\nu/\nu_0 \right|}{\partial T} = \frac{B \cdot C \cdot g(T)}{T^2} \cdot \exp\left(\frac{-C \cdot g(T)}{T}\right) + D \cdot E \cdot T^{(E-1)} \cong 2 \times 10^{-10} \cdot (\text{Hz/Hz})/\text{K} \quad (2b)$$

where  $B$ ,  $C$ , and  $g(T)$  are the coefficients describing the temperature dependence of the skin depth, while  $D$  and  $E$  are the coefficients of thermal lattice expansion. The present work was performed with short term temperature control of 1  $\mu\text{K}$ , and long term stability of about 10  $\mu\text{K}$ . Chui and Lipa<sup>9)</sup> have used paramagnetic salt thermometers in a four stage thermal isolation system to demonstrate temperature stability better than 1 nK. Their system has also been flown in space as part of the Shuttle Lambda Point Experiment program. We propose to use a simplified version of this system to achieve temperature control to 10 nK or better, therefore reducing the temperature induced frequency fluctuations below the  $10^{-18}$  level.

#### D. FLUCTUATIONS IN THE ENERGY STORED IN THE RESONATOR

Fluctuations in the energy  $U$  stored in the resonator will change the electromagnetic radiation pressure on the cavity walls and modify the non-linear superconducting surface reactance. This variation can be expressed as:

$$\left| \frac{\delta\nu}{\nu_0} \right| = -(k_{EM} + k_x) \cdot U \Rightarrow \frac{\partial(\delta\nu/\nu_0)}{\partial U} \cong -1.7 \times 10^{-6} \cdot (\text{Hz/Hz})/\text{J} \quad (3)$$

where the  $k$  coefficients quantify the electromagnetic radiation pressure and the surface reactance. In the present configuration the stored energy in the cavity is  $6 \times 10^{-8}$  J, with a short term stability better than  $10^{-2}$ . Improved electronics will allow both better power control and sensitivity to lower stored energies, therefore reducing the frequency instabilities caused by this effect to the  $10^{-18}$  level.

#### E. COSMIC RADIATION

We consider two cosmic radiation effects: temperature gradients induced by heating due to direct energy deposition, and trapped flux motion caused by local cosmic ray heating. The upper limit for energy deposited by cosmic radiation is about  $10^{-11}$  W/g of material or about  $10^{-10}$  W per cubic centimeter of niobium. The thermal conductivity  $k$  of the very pure niobium used for the resonator is about 0.1 W/cm/K at 1.2K. Using the simple thermal model of a cylinder of radius  $R$ , with constant heating  $Q$  per unit volume,

the temperature increase  $\Delta T$  at the center of the cylinder is given by:  $\Delta T = Q \cdot R^2 / 4k$ . Turning the radiation on/off will thus yield the maximum temperature difference of:  $\Delta T = 1$  nK. This is well below the required stability level of 10 nK discussed in the previous section.

Cosmic rays can cause heating in the vicinity of a flux vortex, causing it to move, and thus modifying the structure of the trapped surface field and shifting the frequency. For high energy cosmic radiation protons, which penetrate the cavity walls, the average expected energy loss in niobium is of the order of 50 MeV/cm. Using the thermal model of an instantaneous line source, the radius of the spot heated to above 5K, (about half the superconducting transition temperature for niobium, where some flux motion is possible) will be about 1  $\mu\text{m}$ . The complete removal (or addition) of a flux vortex to the cavity surface will shift the frequency by about the ratio of the volume of a fluxon, (defined as the fluxon area multiplied by the skin depth), to the cavity volume, or  $|\delta\nu/\nu_0| \cong 1 \times 10^{-18}$  per fluxon. However the probability of flux generation and/or annihilation in the surface of bulk niobium with low impurity levels is extremely small, making this frequency variation mechanism negligible. Motion of the fluxons by  $10^{-4}$  of the cavity size will contribute much less to the frequency variations than flux generation and/or annihilation, and consequently we expect all trapped flux effects under cosmic radiation to be negligible. Experiments with a 1 mCi Cobalt 60 source at 10 cm from the SCSO showed no measurable frequency shifts at the levels  $|\delta\nu/\nu_0| < 1 \times 10^{-15}$ .

#### IV. CONCLUSIONS

SCSO clocks have demonstrated an Allen variance noise floor of  $3 \times 10^{-16}$  for time intervals between 10 and 1000 seconds. The main disturbance effects are due to noise in the electronics and long term drift caused by fluctuations in the temperature, the local gravity, and the electromagnetic energy stored in the resonator. Proposed improvements show the promise of achieving frequency stabilities below  $10^{-17}$  on the ground, and below  $10^{-18}$  for space experiments.

#### ACKNOWLEDGMENTS

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